ECS 452: Exercise 2 Solution

Instructions

1. Separate into groups of no more than three persons.
2. The group cannot be the same as your former group.
3. Only one submission is needed for each group.
4. ***Write down all the steps*** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
5. **Do not panic.**

Name	ID
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1. Consider a DMC whose $\mathcal{X}=\{1,2,3\}$, $\mathcal{Y}=\{1,2,3,4\}$, and $\mathbf{Q}=\begin{bmatrix}0.2 & 0.6 & 0.1 & 0.1\\ 0.1 & 0.7 & 0.1 & 0.1\\ 0.3 & 0.3 & 0.3 & 0.1\end{bmatrix}$.

   Suppose the input probability vector is $\underline{\mathbf{p}}=\begin{bmatrix}0.2 & 0.1 & 0.7\end{bmatrix}$.

   a. Find the joint pmf matrix $\mathbf{P}$.

Multiply each row in the Q matrix by its corresponding $p(x)$

$$Q=\begin{matrix}x\backslash y\\ 1\\ 2\\ 3\end{matrix}\begin{bmatrix}1 & 2 & 3 & 4\\ 0.2 & 0.6 & 0.1 & 0.1\\ 0.1 & 0.7 & 0.1 & 0.1\\ 0.3 & 0.3 & 0.3 & 0.1\end{bmatrix}\begin{matrix}\times 0.2\\ \times 0.1\\ \times 0.7\end{matrix}\quad\begin{matrix}x\backslash y\\ 1\\ 2\\ 3\end{matrix}\begin{bmatrix}1 & 2 & 3 & 4\\ 0.04 & 0.12 & 0.02 & 0.02\\ 0.01 & 0.07 & 0.01 & 0.01\\ 0.21 & 0.21 & 0.21 & 0.07\end{bmatrix}=P$$

   b. Find the MAP detector and its error probability.

$$P=\begin{matrix}x\backslash y\\ 1\\ 2\\ 3\end{matrix}\begin{bmatrix}1 & 2 & 3 & 4\\ 0.04 & 0.12 & 0.02 & 0.02\\ 0.01 & 0.07 & 0.01 & 0.01\\ \mathbf{0.21} & \mathbf{0.21} & \mathbf{0.21} & \mathbf{0.07}\end{bmatrix}$$

For each column of the P matrix, select the max value.

The corresponding x-value for the selected value in each column.

y	$\hat{x}_{MAP}(y)$
1	3
2	3
3	3
4	3

So, $\hat{x}_{MAP}(y)\equiv 3$.

$P(C)=0.21+0.21+0.21+0.07=0.7$

$P(E)=1-P(C)=1-0.7=0.3$

   c. Find the ML detector and its error probability.

For each column of the Q matrix, select the max value.

$$Q=\begin{matrix}x\backslash y\\ 1\\ 2\\ 3\end{matrix}\begin{bmatrix}1 & 2 & 3 & 4\\ 0.2 & 0.6 & 0.1 & \mathbf{0.1}\\ 0.1 & \mathbf{0.7} & 0.1 & \mathbf{0.1}\\ \mathbf{0.3} & 0.3 & \mathbf{0.3} & \mathbf{0.1}\end{bmatrix}\begin{matrix}\times 0.2\\ \times 0.1\\ \times 0.7\end{matrix}\quad\begin{matrix}x\backslash y\\ 1\\ 2\\ 3\end{matrix}\begin{bmatrix}1 & 2 & 3 & 4\\ 0.04 & 0.12 & 0.02 & \mathbf{0.02}\\ 0.01 & \mathbf{0.07} & 0.01 & \mathbf{0.01}\\ \mathbf{0.21} & 0.21 & \mathbf{0.21} & \mathbf{0.07}\end{bmatrix}=P$$

Select the same elements as in the Q matrix.

The corresponding x-value for the selected value in each column

y	$\hat{x}_{ML}(y)$
1	3
2	2
3	3
4	1

All values in the last column are the same. So, we can use any of them.

$P(C)=0.21+0.07+0.21+0.02=0.51$

$P(E)=1-0.51=0.49$

Alternative answer

y	$\hat{x}_{ML}(y)$
1	3
2	2
3	3
4	2

$P(C)=0.21+0.07+0.21+0.01=0.50$

$P(E)=1-0.5=0.5$

Another alternative answer

y	$\hat{x}_{ML}(y)$
1	3
2	2
3	3
4	3

$P(C)=0.21+0.07+0.21+0.07=0.56$

$P(E)=1-0.56=0.44$